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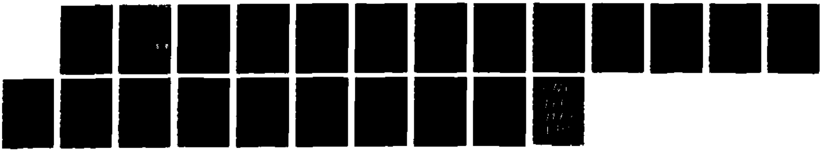

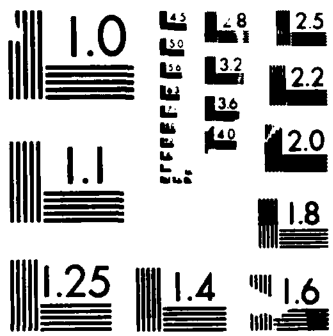

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STABILITY ENHANCEMENT OF FLEXIBLE STRUCTURES BY NONLINEAR BOUNDARY-FEEDBACK CONTROL

A.V. Balakrishnan

ABSTRACT

We establish strong stability for a class of nonlinear boundary feedback controllers using an abstract wave-equation formulation of a beam stabilization problem arising in the control of flexible structures in space.

Paper presented at the IFIP Working Conference on "Boundary Control and Boundary Variations," Nice, June 1986

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1. Introduction

The problem of active feedback stabilizing flexible structures is of recent interest especially for deployment in space [1]. A significant feature of this application is the need for "robust" controllers - whose design does not require precise knowledge of system parameters -- see [2]. Although a class of such linear stabilizing control laws is given in [2], it would appear that to generate the control effort necessary, the actuators (reaction jets, for example), would need to be nonlinear: linear for small amplitudes but nonlinear (saturating-type) for large amplitudes.

Another feature of the space application is the uncertainty in the damping parameters (even including the model). The strong stabilizability using linear controllers requires controllability and this was established in [2] neglecting damping; but the proof of controllability when damping is present is unclear. In this paper we show that for a reasonable damping model the robust linear controller still yields strong stability under a natural extension of a condition involving the undamped modes used in [2]. Under the asumption that there is nonzero natural damping, however small, we prove that we have strong stability for a class of nonlinear feedback control laws which are linear for small amplitudes and can saturate for large amplitudes.

We only treat the abstract wave-equation version of the problem: the reduction of the physical model (biharmonic beam equations with delta-function controls) to the abstract form is given in [3]. We only note here one feature of the problem: the differential operators do not involve any "boundary conditions": and although the control is exerted on the boundary, the formulation does not fall into the class of boundary-control problems treated by

Lasiecka and Triggiani [4] so that in particular the results therein are not directly applicable.

Finally we note that our results may be regarded as an extension of the Benchimol-Slemrod result [5] (see Levan [6] for a recent treatment) to a class of nonlinear controllers but without invoking controllability -- albeit in a particular case. Our proof, although totally elementary, relies heavily on the eigenfunction decomposition of the generator. The problem statement and the main results are in Section 2.

2. General Results

Let $\mathcal{H}$ denote a separable Hilbert space and let us consider the following canonical abstract differential equation characterizing the response $x(t)$ of a flexible structure to the applied input $u(t)$:

$$M\ddot{x}(t) + D\dot{x}(t) + Ax(t) + Bu(t) = 0 \tag{2.1}$$

where M is a self-adjoint positive definite (zero is in the resolvent spectrum) linear bounded operator mapping $\mathcal{H}$ into $\mathcal{H}$;

A is a self-adjoint nonnegative definite closed linear operator with dense domain and with compact resolvent; we shall (for simplicity) also assume that zero is in the resolvent set.

D is a self-adjoint nonnegative definite closed linear operator whose domain includes that of $\sqrt{A}$; and

B is a finite-dimensional linear operator mapping R^m into $\mathcal{H}$.

The next step is to introduce the "energy norm" space, which we shall denote by $\mathcal{H}_E$. On the product space

$$\mathcal{D}(\sqrt{A}) \times \mathcal{H} \tag{2.2}$$

we can introduce the "energy" inner product:

$$[Y,Z]_E = [\sqrt{A}\, y_1, \sqrt{A}\, z_1] + [My_2, z_2] \tag{2.3}$$

where

$$Y = \begin{vmatrix} y_1 \\ y_2 \end{vmatrix} \quad ; \qquad Z = \begin{vmatrix} z_1 \\ z_2 \end{vmatrix} \tag{2.4}$$

and complete the space in this inner product to yield $\mathcal{H}_E$. To avoid confusion we shall use a subscript E to denote the inner product in $\mathcal{H}_E$. Let $\mathcal{A}$ denote the operator

$$\mathcal{A} = \begin{vmatrix} 0 & I \\ -M^{-1}A & -M^{-1}D \end{vmatrix} \tag{2.5}$$

with domain:

$$\mathcal{D}(\mathcal{A}) = \left[Y \,\middle|\, Y = \begin{vmatrix} y_1 \\ y_2 \end{vmatrix} ; \quad \begin{matrix} y_1 \in \mathcal{D}(A) \\ y_2 \in \mathcal{D}(\sqrt{A}) \end{matrix} \right] . \tag{2.6}$$

We shall now show that $\mathcal{A}$ is closed.

$$\begin{vmatrix} y_{1,n} \\ y_{2,n} \end{vmatrix} = Y_n \in \mathcal{D}(\mathcal{A}) \; ; \qquad Y_n \to Y = \begin{vmatrix} y_1 \\ y_2 \end{vmatrix}$$

$$\begin{vmatrix} z_{1,n} \\ z_{2,n} \end{vmatrix} = Z_n = \mathcal{A}Y_n \; ; \qquad Z_n \to Z = \begin{vmatrix} z_1 \\ z_2 \end{vmatrix}$$

where

$$z_{1,n} = y_{2,n} \tag{2.7}$$

$$Mz_{2,n} = -Ay_{1,n} - Dy_{2,n} \tag{2.8}$$

and

$$\|z_n - z\|_E^2 = \|\sqrt{A}\,(z_{1,n} - z_1)\|^2 + [(M(z_{1,n} - z_n),\ (z_{2,n} - z_n)]$$

Since $(\sqrt{A})$ has a bounded inverse, it follows that

$$z_{1,n} = (\sqrt{A})^{-1}(\sqrt{A}\, z_{1,n})$$

converges. But by (2.7)

$$y_{2,n} = z_{1,n}$$

so that

$$\{y_{2,n}\} \quad \text{and} \quad \{\sqrt{A}\, y_{2,n}\}$$

are Cauchy. Now since domain of D includes that of $\sqrt{A}$

$$D(\sqrt{A})^{-1}$$

is linear bounded, and hence

$$Dy_{2,n} = D(\sqrt{A})^{-1}(\sqrt{A}\, y_{2,n})$$

converges. Since the left side of (2.8) is Cauchy, this implies that

$$\{Ay_{1,n}\}$$

is Cauchy, and of course $\{y_{1,n}\}$ is Cauchy. Hence we see that $\mathcal{A}$ is closed.

Since

$$\mathcal{A}^* = \begin{bmatrix} 0 & -I \\ M^{-1}A & -M^{-1}D \end{bmatrix}$$

with same domain as $\mathcal{A}$, and for Y in $\mathcal{D}(\mathcal{A})$:

$$[\mathcal{A}Y, Y]_E + [\mathcal{A}^*Y, Y]_E = -2[Dy_2, y_2] \tag{2.9}$$

where

$$Y = \begin{vmatrix} y_1 \\ y_2 \end{vmatrix}$$

we see that $\mathcal{A}$ and $\mathcal{A}^*$ are dissipative, and hence [7] $\mathcal{A}$ generates a strongly continuous contraction semigroup which we shall denote $S(t)$, $t \geq 0$. Let $\mathcal{A}_0$ denote the "undamped" generator:

$$\mathcal{A}_0 = \begin{bmatrix} 0 & I \\ -M^{-1}A & 0 \end{bmatrix}$$

with domain

$$= \left[Y \;\middle|\; Y = \begin{vmatrix} y_1 \\ y_2 \end{vmatrix}, \quad y_1 \in \mathcal{D}(A) \right] .$$

Let

$$\mathcal{D} = \begin{bmatrix} 0 & 0 \\ 0 & M^{-1}D \end{bmatrix} .$$

We note that $\lambda > 0$ belongs to the resolvent of $\mathcal{A}_0$ and that $R(\lambda, \mathcal{A}_0)$ is compact. From the easily verified resolvent equation:

$$R(\lambda, \mathcal{A}) = R(\lambda, \mathcal{A}_0)\,[I + \mathcal{D}R(\lambda, \mathcal{A})] \tag{2.10}$$

it follows that $R(\lambda, \mathcal{A})$ is compact, since $\mathcal{D}R(\lambda, \mathcal{A})$ is bounded. Rewriting (2.1) as

$$\dot{Y}(t) = \mathcal{A}Y(t) + \mathcal{B}u(t) \tag{2.11}$$

where

$$Y(t) = \begin{vmatrix} x(t) \\ \dot{x}(t) \end{vmatrix}$$

$$\mathcal{B}u = \begin{vmatrix} 0 \\ -M^{-1}Bu \end{vmatrix}$$

we see that (2.11) has the "mild" solution:

$$Y(t) = S(t)\, Y(0) + \int_0^t S(t-\sigma)\, \mathcal{B}u(\sigma)\, d\sigma \tag{2.12}$$

Let $S_0(\cdot)$ denote the semigroup generated by A_0. Let $\{\phi_k\}$ denote the eigenvectors of

$$M^{-1}A\, \phi_k = \omega_k^2\, \phi_k$$

orthonormalized so that

$$[M\phi_k, \phi_j] = \delta_j^k$$

and such that ω_k^2 are monotone increasing. Note that these are the undamped or "natural" modes.

We shall say that a (time-invariant) "feedback" control

$$u(t) = K(Y(t)) \tag{2.13}$$

where $K(\cdot)$ maps H_E into R^m, "stabilizes" the system (2.11) if

$$Y(t) = S(t)\, Y(0) + \int_0^t S(t-\sigma)\, \mathcal{B}K(Y(\sigma))\, d\sigma$$

has a unique strongly continuous solution such that it is globally stable. That is to say

$$\|Y(t)\|_E \to 0 \qquad \text{as} \quad t \to \infty$$

for every initial $Y(0)$.

We begin with linear controllers.

THEOREM 2.1. Suppose

$$[\phi_k, (D + BB^*)\phi_k] \neq 0 \tag{2.14}$$

for every k. Then the linear feedback control

$$u(t) = -PB^*Y(t) \tag{2.15}$$

where P is positive definite and has a bounded inverse, yields global asymptotic stability.

Conversely, (2.14) is necessary if these controls are to yield global stability.

Proof. To begin with we assume that the strong version of (2.11) is satisfied:

$$M\ddot{x}(t) + D\dot{x}(t) + Ax(t) + Bu(t) = 0 \ . \tag{2.16}$$

Let

$$E(t) = \frac{1}{2} \|Y(t)\|_E^2$$

$$= \frac{1}{2} ([\sqrt{A}\, x(t), \sqrt{A}\, x(t)] + [M\dot{x}(t), \dot{x}(t)]) \ . \tag{2.17}$$

Then

$$\frac{d}{dt} E(t) = -[D\dot{x}(t), \dot{x}(t)] - [\dot{x}(t), Bu(t)] \tag{2.18}$$

$$= -[(D + \lambda(t)BB^*)\dot{x}(t), \dot{x}(t)] \tag{2.19}$$

where

$$\lambda(t) = \frac{[u(t), B^*\dot{x}(t)]}{\|B^*\dot{x}(t)\|^2} \tag{2.20}$$

$$= 0 \qquad \text{if} \quad \|B^*\dot{x}(t)\| = 0 \ .$$

From (2.18) we obtain that

$$E(t) \quad \text{is monotone nonincreasing in} \quad t$$

and that

$$E(0) - E(\infty) = \int_0^\infty [D\dot{x}(t), \dot{x}(t)]\, dt + \int_0^\infty \lambda(t)[B^*\dot{x}(t), B^*\dot{x}(t)]\, dt \quad . \tag{2.21}$$

Hence if we consider first the choice

$$u(t) = B^*\dot{x}(t) \tag{2.22}$$

or equivalently

$$\lambda(t) = 1$$

we have correspondingly

$$\int_0^\infty [D\dot{x}(t), \dot{x}(t)]\, dt + \int_0^\infty [B^*\dot{x}(t), B^*\dot{x}(t)]\, dt = E(0) - E(\infty) < \infty \, . \tag{2.23}$$

To prove global stability, it is enough to show that

$$E(\infty) = 0$$

in (2.21), or in (2.23). Next let

$$a_k(t) = [x(t), M\phi_k]$$

so that we have the "modal" expansion:

$$x(t) = \sum_1^\infty a_k(t)\, \phi_k \tag{2.24}$$

and

$$E(t) = \frac{1}{2} \sum_1^\infty \left(\omega_k^2 a_k(t)^2 + \dot{a}_k(t)^2\right) \quad . \tag{2.25}$$

Using this expansion in (2.16) we have

$$\ddot{a}_k(t) + \omega_k^2 a_k(t) = -[D\dot{x}(t), \phi_k] - [B^*\dot{x}(t), B^*\phi_k] \tag{2.26}$$

or,

$$\ddot{a}_k(t) + \{[D\phi_k, \phi_k] + [B^*\phi_k, B^*\phi_k]\}\dot{a}_k(t) + \omega_k^2 a_k(t) = -f_k(t) \tag{2.27}$$

where

$$f_k(t) = [D\dot{x}(t), \phi_k] - \dot{a}_k(t)[D\phi_k, \phi_k] + [B^*\dot{x}(t), B^*\phi_k] - \dot{a}_k(t)[B^*\phi_k, B^*\phi_k] .$$

Suppose now that for some k:

$$D\phi_k = 0 ; \qquad B^*\phi_k = 0 .$$

Then

$$x(t) = a_k(t)\phi_k$$

is a solution of (2.17), provided only that

$$\ddot{a}_k(t) + \omega_k^2 a_k(t) = 0$$

and hence $|a_k(t)|$ does <u>not</u> go to zero as $t \to \infty$; this takes care of the necessity condition. Assume then that

$$2\sigma_k = [D\phi_k, \phi_k] + [B^*\phi_k, B^*\phi_k]$$

is nonzero. Then (2.27) can be "solved" to yield (assuming small damping for simplicity):

$$a_k(t) = a_k e^{-\sigma_k t} \cos \lambda_k t + b_k e^{-\sigma_k t} \frac{\sin \lambda_k t}{\lambda_k} - \int_0^t W(t-s) f_k(s)\, ds \tag{2.28}$$

where

$$W(t) = \frac{e^{-\sigma_k t} \sin \lambda_k t}{\lambda_k} ;$$

$(-\sigma_k \pm i\lambda_k)$ are the roots of

$$s^2 + 2\sigma_k s + \omega_k^2 = 0 .$$

Now because:

$$|[D\dot{x}(t), \phi_k]|^2 \leqq [D\dot{x}(t), \dot{x}(t)] [D\phi_k, \phi_k]$$

it follows from (2.23) that

$$\int_0^\infty |[D\dot{x}(t), \phi_k]|^2 \, dt < \infty .$$

Hence a little analysis shows that

$$\int_0^t W(t-s) [D\dot{x}(s), \phi_k] \, ds \to 0 \qquad \text{as} \quad t \to \infty .$$

By a similar reasoning, also

$$\int_0^t W(t-s) [B^*\dot{x}(s), B^*\phi_k] \, ds \to 0 \qquad \text{as} \quad t \to \infty .$$

Hence it follows that

$$\varepsilon(t) = a_k(t) - 2\sigma_k \int_0^t W(t-s) \dot{a}_k(s) \, ds \tag{2.29}$$

where

$$\varepsilon(t) \to 0 \qquad \text{as} \quad t \to \infty .$$

By integration by parts we obtain that

$$\varepsilon(t) = a_k(t) + a_k(0) W(t) + 2\sigma_k \int_0^t \dot{W}(t-\sigma) \, a_k(\sigma) \, d\sigma .$$

Hence

$$a_k(t) + 2\sigma_k \int_0^t \dot{W}(t-\sigma)\, a_k(\sigma)\, d\sigma = \beta(t) \tag{2.30}$$

where

$$\beta(t) \to 0 \qquad \text{as} \quad t \to \infty \; .$$

Solving (2.30) (using elementary Laplace transform techniques) we obtain that

$$a_k(t) \to 0 \qquad \text{as} \quad t \to \infty \; .$$

Differentiating (2.28) and proceeding in a similar way we can also show that

$$\dot{a}_k(t) \to 0 \qquad \text{as} \quad t \to \infty \; .$$

Let

$$\Phi_k^+ = \begin{vmatrix} \phi_k \\ i\omega_k \phi_k \end{vmatrix}$$

$$\Phi_k^- = \begin{vmatrix} \phi_k \\ -i\omega_k \phi_k \end{vmatrix} \; .$$

Then

$$\lim_{t\to\infty} [Y(t),\, \Phi_k^+] = \lim_{t\to\infty} [Y(t),\, \Phi_k^-] = 0$$

for each k, which is enough to imply that $Y(t)$ converges weakly to zero. Next we note that

$$Y(t) = S_B(t)\, Y(0)$$

where $S_B(t)$ is the (strongly continuous contraction) semigroup generated by

$$\mathcal{A} - \mathcal{B}\mathcal{B}^*$$

which has a compact resolvent. Hence weak stability implies strong stability [7]:

$$\|Y(t)\|_E = \|S_B(t)\ Y(0)\|_E \to 0 . \tag{2.31}$$

In particular we have (2.23) that

$$\|Y(0)\|_E^2 = \int_0^\infty [D\dot{x}(t), \dot{x}(t)]\ dt + \int_0^\infty \|B^*\dot{x}(t)\|^2\ dt$$

or,

$$\|Y(0)\|_E^2 = \int_0^\infty [\mathcal{D}Y(t), Y(t)]_E\ dt + \int_0^\infty \|\mathcal{B}^*Y(t)\|^2\ dt . \tag{2.32}$$

Finally given any arbitrary initial condition $Y(0)$, we can find an approximating sequence $\{Y_n(0)\}$ in the domain of $\mathcal{A}$ such that (2.31) holds for the corresponding solution $Y_n(t)$; and the result follows from the estimate:

$$\|Y(t)\|_E \leq \|S_B(t)\ Y_n(0)\|_E + \|Y_n(0) - Y(0)\|_E .$$

REMARK 1. We note that if $D = 0$, our condition (2.14) is equivalent to requiring $(A \sim B)$ controllability (see [2]). Hence we would have strong stability by a more general argument due to Benchimol [5]. If $B = 0$, then condition (2.14) implies that $D\phi_k \neq 0$ for any k, and we are then proving strong stability for the semigroup $S(t)$.

Nonlinear Controllers

Let us now go on to consider a class of nonlinear controllers. Thus we shall consider where $K(\cdot)$ in (2.13) is given by

$$K(Y) = f(B^*y_2)$$

where

$$Y = \begin{vmatrix} y_1 \\ y_2 \end{vmatrix} ,$$

and $f(\cdot)$ maps R^m into R^m, and satisfies the following conditions:

i) $[f(u), u] > 0$ for $u \neq 0$

ii) $\|f(u)\| \leq \lambda\|u\|$

iii) $f(\cdot)$ is Lipschitz.

A typical example of $f(\cdot)$ is

$$f(u) = v ; \qquad u = \{u_i\} ; \qquad v = \{v_i\}$$

$$v_i = \gamma_i \tan^{-1} \mu_i u_i ; \qquad \gamma_i, \mu_i > 0$$

THEOREM 2.2. Suppose

$$[D\phi_k, \phi_k] \neq 0 \tag{2.14a}$$

for every k. Then the feedback control

$$u(t) = -B^* f(B^*\dot{x}(t)) \tag{2.33}$$

where

$$Y(t) = \begin{vmatrix} x(t) \\ \dot{x}(t) \end{vmatrix}$$

yields asymptotic global stability.

Proof. Under condition (2.14a), the semigroup $S(\cdot)$ generated by A

in $\mathcal{H}_E$ is strongly stable. By virtue of the Lipschitz conditions on $f(\cdot)$, existence and uniqueness of solution for each $t > 0$ is immediate, and the solution is given by:

$$Y(t) = S(t)\,Y(0) + \int_0^t S(t-\sigma)\,\mathcal{B}\,f(B^*\dot{x}(\sigma))\,d\sigma, \qquad 0 < t . \tag{2.34}$$

Next we shall show that

$$\int_0^\infty \|f(B^*\dot{x}(\sigma))\|^2\,d\sigma < \infty . \tag{2.35}$$

For this purpose let us assume first that the initial condition $Y(0)$ is in the domain of the generator $\mathcal{A}$, so that we have

$$M\ddot{x}(t) + D\dot{x}(t) + Bf(B^*\dot{x}(t)) + Ax(t) = 0 .$$

Defining the energy again as

$$E(t) = \frac{1}{2}\,\|Y(t)\|_E^2$$

we have that

$$\dot{E}(t) = -[D\dot{x}(t), \dot{x}(t)] - [\dot{b}(t), f(\dot{b}(t))]$$

where

$$b(t) = B^*x(t) .$$

Since

$$[\dot{b}(t), f(\dot{b}(t))] \geq 0 ,$$

we have that:

$$\int_0^\infty [D\dot{x}(t), \dot{x}(t)]\,dt < \infty$$

and since we can write

$$(\sqrt{D})^{-1}(\sqrt{D}\,\dot{x}(t)) = \dot{x}(t)$$

it follows that

$$\int_0^\infty \|\dot{x}(t)\|^2\,dt < \infty .$$

But this implies that

$$\int_0^\infty \|B^*\dot{x}(t)\|^2\,dt < \infty$$

and hence also

$$\int_0^\infty \|f(B^*\dot{x}(t))\|^2\,dt < \infty$$

by virtue of our assumptions on $f(\cdot)$.

We now proceed as in the proof of the preceding theorem. Writing

$$2\sigma_k = [D\phi_k, \phi_k]$$

we have:

$$\ddot{a}_k(t) + \omega_k^2 a_k(t) + 2\sigma_k \dot{a}_k(t) = -f_k(t) \tag{2.36}$$

where $a_k(t)$ is as before, and

$$f_k(t) = [D\dot{x}(t), \phi_k] - \dot{a}_k(t)[D\phi_k, \phi_k] + [f(B^*\dot{x}(t)), B^*\phi_k]$$

since

$$\int_0^\infty [D\dot{x}(t), \phi_k]^2\,dt + \int_0^\infty [f(B^*\dot{x}(t)), B^*\phi_k]^2\,dt < \infty ,$$

as before, we obtain

$$\varepsilon(t) = a_k(t) + a_k(0)\,w(t) + 2\sigma_k \int_0^t \dot{w}(t-\sigma)a_k(\sigma)\,d\sigma$$

where $\varepsilon(t) \to 0$ as $t \to \infty$ and the rest of the arguments follow. Hence

$$a_k(t) \to 0 \qquad \text{as} \quad t \to \infty$$

$$\dot{a}_k(t) \to 0 \qquad \text{as} \quad t \to \infty$$

for each k. Hence it follows that

$$Ax(t) \rightharpoonup 0$$

$$A\dot{x}(t) \rightharpoonup 0 \,.$$

Since A^{-1} is compact, it follows that $x(t)$ converges strongly to zero (in $\mathcal{H}$). Similarly

$$A\dot{x}(t) \rightharpoonup 0$$

implies that $\dot{x}(t)$ converges strongy to zero (in $\mathcal{H}$). Hence $Y(t)$ converges weakly (in $\mathcal{H}_E$) to zero. Now

$$[D\dot{x}(t), \phi_k] = [\dot{x}(t), D\phi_k]$$

implies that

$$D\dot{x}(t) \rightharpoonup 0 \,.$$

Hence

$$\mathcal{A}Y(t) \rightharpoonup 0 \,.$$

since $\mathcal{A}$ has a compact resolvent it follows that $Y(t)$ converges strongly (In $\mathcal{H}_E$) to zero.

Next we relax our assumption regarding the initial condition $Y(0)$. We begin by proving Lipschitz continuity of the solution with respect to the initial condition. Thus let $Y(t)$ denote the solution with initial condition $Y(0)$ and let

$$Y(t) = M(t)(Y(0)) .$$

Then let $Y_1(0), Y_2(0) \in \mathcal{H}_E$ and

$$M(t)(Y_2(0)) - M(t)(Y_1(0)) = S_D(t)(Y_2(0) - Y_1(0))$$

$$+ \int_0^t S_D(t-\sigma)\left(\mathcal{B}f(\mathcal{B}^*Y_2(\sigma)) - \mathcal{B}f(\mathcal{B}^*Y_1(\sigma))\right) d\sigma .$$

Let

$$m(t) = \|Y_2(t) - Y_1(t)\|_E .$$

Then in view of our assumptions on $f(\cdot)$, we have

$$m(t) \leq m(0) + \gamma\int_0^t m(\sigma)\, d\sigma ; \qquad \gamma \geq 0$$

and hence by the usual analysis

$$m(t) \leq e^{\gamma t} m(0)$$

yielding Lipschitz continuity. The continuity yields in turn

$$\|Y(t)\|_E^2 = 2E(t) = \|Y(0)\|_E^2 - \int_0^t [D\dot{x}(s), \dot{x}(s)]\, ds$$

$$- \int_0^t [\dot{b}(s), f(\dot{b}(s))]\, ds .$$

and hence we obtain:

$$\int_0^\infty \|D\dot{x}(t)\|^2\, dt + \int_0^\infty [\dot{b}(t), f(\dot{b}(t))]\, dt < \infty .$$

Next we need to establish (2.36). But this follows readily from the fact (2.11) holds in the weak sense. Hence we obtain that $Y(t)$ converges strongly to zero in $\mathcal{H}_E$ for any initial $Y(0)$ in $\mathcal{H}_E$.

REMARK. It would be of interest to establish strong stability under the weaker condition (2.14). We note that the condition (2.14) is again obviously necessary.

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